# Investigations of Heat Transfer and Pressure Drop Between Parallel Channels with Pseudoplastic and Dilatant Fluids

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Received 13 February 2002; accepted 15 January 2003

**ABSTRACT:** Central to the problem of heat exchangers design is the prediction of pressure drop and heat transfer in the noncircular exchanger duct passages such as parallel channels. Numerical solutions for laminar fully developed flow are presented for the pressure drop (friction factor times Reynolds number) and heat transfer (Nusselt numbers) with thermal boundary conditions [constant heat flux (CHF) and constant wall temperature (CWT) ] for a pseudoplastic and dilatant non-Newtonian fluid flowing between infinite parallel channels. A shear rate parameter could be used for the prediction of the shear rate range for a specified set of operating conditions that has Newtonian behavior at low shear rates, power law behavior at high shear rates, and a transition region in between. Numerical results of the Nusselt number [constant heat flux (CHF) and constant wall temperature (CWT) ] and the product of the friction factor and Reynolds number for the Newtonian region were compared with the literature values showing agreement within 0.36% in the Newtonian region. For pseudoplastic and dilatant non-Newtonian fluids, the modified power law model is recommended to use because the fluid properties have big discrepancies between the power law model and the actual values in low and medium range of shear rates.  J Appl Polym Sci 89: 3601–3608, 2003

**Key words:** modified power law; non-Newtonian fluid; pseudoplastic fluid; dilatant fluid; shear rate parameter; parallel channel

## INTRODUCTION

In a number of industries, such as the chemical, electromechanical, pharmaceutical, biotechnological, hemodynamics, polymer bionanocomposites, and food industries, it is usual to treat with non-Newtonian fluids. Many non-Newtonian fluids have viscous properties that are different in the various shear rate ranges. Non-Newtonian fluids may be defined as fluids for which the flow curve ($\tau$ vs $\gamma$) is not linear through the origin at a given temperature and pressure.

Irvine and Karni[1] provided a general overview of non-Newtonian fluids, discussing rheological property measurements, pressure drop, and heat transfer. A large number of constitutive equation have been developed to describe the behavior of purely viscous non-Newtonian fluids (Skelland[2]). Some have as many as five rheological properties, and while they are suitable for describing in detail the relations between shear stress and shear rate for complex fluids, they are normally too cumbersome to use in engineering analyses.

Central to the problem of heat exchangers design is the prediction of pressure drop and heat transfer in the noncircular exchanger duct passages such as parallel channels. For fully developed laminar flow of Newtonian and non-Newtonian power law fluids in a circular duct, the solutions are well known for both the classical boundary conditions of constant wall temperature (CWT) and constant wall heat flux (CHF) and the pressure drop.

For Newtonian fluids, pressure drop and heat transfer coefficients were calculated by Shah and London,[3] Rothfus et al.,[4] etc. For power law fluids, Chandrupatla,[5] Wheeler and Wissler,[6] Kozicki and Tiu,[7] Kozicki et al.,[8] Hartnett et al.,[9] Hartnett and Kostic,[10] Pinho and Whitelaw,[11] and Lin et al.[12] obtained those analytically and experimentally.

For power law fluids, such solutions are also available (Beek and Eggink[13] and Grigull[14]). For situations where the velocity profile is fully developed but the temperature profile is developing (the Graetz problem), solutions have also been reported for Newtonian fluids for both boundary conditions (Shah and London[3]) and for power law fluids (Bird and Hassager,[15]

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Journal of Applied Polymer Science, Vol. 89, 3601–3608 (2003)

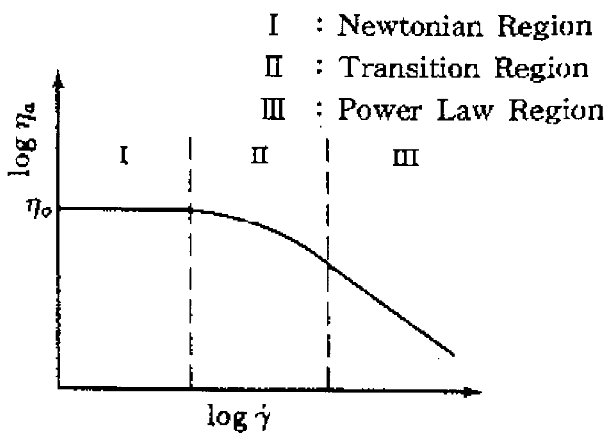


**Figure 1** Typical flow curve of pseudoplastic fluid.

Lyche and Bird,[16] Whitman and Drake,[17] Mckillop,[18] Joshi and Bergles[19])

Recently a large number of heat exchangers is designed and manufactured for the automotive and chemical process industries to heat or cool pseudoplastic and dilatant fluids. Even today, there is a general lack of experimental data for heat transfer coefficients required for the heat exchanger designs. It is felt, however, that the rheological behavior can best be investigated with a well-defined geometry often found in industry, such as a parallel duct.

Although a power law model has been used extensively for calculating velocity profile and heat transfer coefficient in engineering, it has significant disadvantages that it only applies to the power law region in the flow curve in Figure 1 and the apparent viscosity (Irvine and Karni[1]) at the centeroid of the duct becomes infinite.

The purpose of the present study is to extend our knowledge and correct this situation by presenting solutions for laminar fluids having the rheological characteristics illustrated in Figure 1 and to develop the relationships between the friction factor–Reynolds number and the heat transfer coefficients for pseudoplastic and dilatant fluids. Such a solution should have the characteristics that at low velocities (low shear rates) the Newtonian solution is an asymptote while at large shear rates the power law solution is an asymptote. In addition, the solution should predict the appropriate pressure drop and heat transfer behavior in the transition zone. Finally, a parameter is needed to predict the shear rate range in terms of the operating characteristics of the system. For a circular tube (Brewster and Irvine[20]), and concentric annulus (Capobianchi and Irvine[21]), such solutions are available.

## THEORETICAL BACKGROUND

A number of constitutive equation can describe the apparent viscosity-shear rate relation (Irvine and Karni[1]) for fluids such as shown in Figure 1. A convenient and useful equation is the "modified power law model," which to the authors' knowledge was first used by Dunleavy and Middleman[22]:

$$\eta_a = \frac{\eta_0}{1 + (\eta_0/K)\dot{\gamma}^{1-n}} \tag{1}$$

Inspection of eq. (1) reveals that at low rates ($\eta_0/K\gamma^{1-n} \gg 1$) the apparent viscosity becomes equal to $\eta_0$ and the fluid is operating in the Newtonian region of Figure 1. At higher shear rates ($\eta_0/K\gamma^{1-n} \gg 1$) the fluid becomes a power law fluid where $\nu_a = K\gamma^{1-n}$. At intermediate shear rates, there is a transition zone. An additional advantage of the modified power law over constitutive equations such as Ellis, Sutterby, Cross, etc., is that the familiar Newtonian and power law Reynolds numbers are retained in the analysis.

The problem facing the designer when considering pseudoplastic and dilatant fluids is which of the three regions shown in Figure 1 will be the operating region for the system under consideration. For low Reynolds number forced flows, the system may well be operating in regions I and II, and even though the fluid is pseudoplastic or dilatant, the power law solution becomes irrelevant and Newtonian and transitional solutions are required. In order to determine the correct shear rate region for a given set of operating conditions a dimensionless shear rate parameter is required. Such a parameter arises naturally by using the modified power law constitutive equation in solving the dimensionless momentum equation for forced convection flows.

The derivation of the shear rate parameter will be illustrated by considering a simple but fundamental case, i.e., fully developed laminar flow of a modified power law fluid between infinite parallel channels. This shear rate parameter should contain those quantities that are under the control of the designer—namely, the properties of the fluid, $K$, $\eta_0$, $n$ and the geometry of the system (the duct diameter), and the average flow velocity $\bar{u}$.

### Pressure drop

It is convenient to start with the conservation equations to solve a problem related to fluid flowing through parallel channel. For steady flow of an incompressible fluid with negligible viscous dissipation, the governing equations depends on the apparent viscosity that is related to the shear stress and shear rate.

An understanding of fully developed flow in parallel channels can be obtained by examining a simple flow situation. Consider the flow between two infinite parallel channels as shown in Figure 2 but with both plates stationary and the fluid being pumped through.

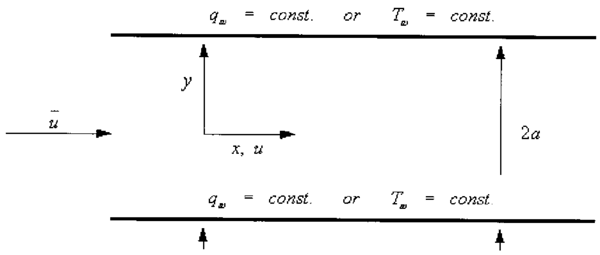


**Figure 2** Coordinate system for parallel channels.

Also consider the coordinate system origin as being midway between the two plates which are a distance $2a$ apart. Using the shear law, $\tau = \eta_a \gamma$, and the appropriate forms of momentum equation, the fully developed shear stress field is described by the following:

$$\frac{d}{dy}\left(\eta_a \frac{du}{dy}\right) = -\frac{dp}{dx} \tag{2}$$

with boundary conditions

$$u(a) = 0, \quad u'(0) = 0 \tag{3}$$

The following dimensionless quantities may be defined:

$$y^+ = \frac{y}{a}, \quad f_a = \frac{2a(dp/dx)}{\rho \bar{u}^2} \tag{4}$$

$$u^+ = \frac{u}{\bar{u}}, \quad \mathrm{Re}_g = \frac{\rho \bar{u}^{2-n} a^n}{K} \tag{5}$$

$$\mathrm{Re}_a = \frac{\rho \bar{u} a}{\eta_0}, \quad \mathrm{Re}_M = \frac{\rho \bar{u} a}{\eta^*} \tag{6}$$

$$\eta^* = \frac{\eta_0}{1+\beta}, \quad \eta_a^+ = \frac{1+\beta}{1+\beta(du^+/dy^+)^{1-n}} \tag{7}$$

$$\beta = \frac{\mathrm{Re}_g}{\mathrm{Re}_a} = \frac{\eta_0}{K}\left(\frac{\bar{u}}{a}\right)^{1-n}, \quad u^{++} = \frac{u^+}{(f_a \cdot \mathrm{Re}_M)/2} \tag{8}$$

$$\mathrm{Re}_M = \mathrm{Re}_a + \mathrm{Re}_g = \frac{\rho \bar{u} a}{\eta_0} + \frac{\rho \bar{u}^{2-n} a^n}{K} = \frac{\rho \bar{u} a}{\eta_0}(1+\beta) \tag{9}$$

$$\eta_a^+ = \frac{1+\beta}{1+\beta(f_a \mathrm{Re}_g/2)^{1-n}(du^{++}/dy^+)^{1-n}} \tag{10}$$

It is of interest to investigate the characteristics of the modified Reynolds number, $\mathrm{Re}_M$, and the shear rate parameter $\beta$. The shear rate between the parallel channels is proportional to $\bar{u}/a$ as is $\beta$. Thus low shear rates occur when $\beta$ is small and high shear rates when $\beta$ is large.

The parameter $\beta$ is the shear rate parameter that determines whether the fluid system is operating in the Newtonian, transition, or power law regions. As $\beta$ becomes small, $\mathrm{Re}_M$ approaches the Newtonian Reynolds number $\mathrm{Re}_a$ and as $\beta$ becomes large, $\mathrm{Re}_M$ approaches the power law Reynolds number $\mathrm{Re}_g$.

For a Newtonian fluid, the continuity equation is

$$\bar{u} = \frac{1}{A_c}\int_{A_c} u\,dA_c = \frac{1}{a}\int_0^a u\,dy \tag{11}$$

The dimensionless form of eq. (12) becomes

$$\int_0^1 u^+ dy^+ = 1 \tag{12}$$

Since $\eta_a = \eta_0$, eq. (3) becomes

$$\frac{d^2u^+}{dy^{+2}} = -\frac{f_a \mathrm{Re}_M}{2}, \quad \frac{d^2u^{++}}{dy^{+2}} = -1 \tag{13}$$

Equation (13) becomes

$$\int_0^1 u^{++} dy^+ = \frac{2}{f_a \mathrm{Re}_M} \tag{14}$$

For a modified power law fluid, eq. (3) may be written in dimensionless form as

$$\frac{d}{dy^+}\left(\eta_a^+ \frac{du^{++}}{dy^+}\right) = -1 \tag{15}$$

with boundary conditions

$$u^{++}(1) = 0, \quad u^{++'}(0) = 0 \tag{16}$$

The continuity equation remains the same as eq. (14).

Therefore, eq. (15) becomes the Newtonian fluid momentum equation when $\beta$ is small and the power law fluid momentum equation when $\beta$ is large. Thus eq. (15) could give the complete solution for the fluids in Figure 1 and the final results can be presented as the product of $f\mathrm{Re}_M$vs the shear rate parameter $\beta$.

### Heat transfer

The energy equation for the thermally developed flow in a fluid between infinite parallel channels neglecting viscous dissipation and rate of energy generation (Incropera and DeWitt[23]) can be written as

$$k\frac{\partial^2 T}{\partial y^2} = \rho c_p u \frac{\partial T}{\partial x} \tag{17}$$

with boundary conditions

$$T(a) = T_w, \quad T'(0) = 0 \tag{18}$$

Consider the case of CHF per unit area at the wall to a fluid between infinite parallel channels.

The temperature field is fully developed when

$$\frac{\partial}{\partial x}\left(\frac{T - T_w}{T_b - T_w}\right) = 0 \tag{19}$$

where $h$ is constant.

From $q_w = h(T_w - T_b)$ = constant,

$$\frac{dT_w}{dx} = \frac{dT_b}{dx} \tag{20}$$

From Eq. (20),

$$\frac{\partial T}{\partial x} = \frac{dT_w}{dx} = \frac{dT_b}{dx} \tag{21}$$

Defining a dimensionless temperature, $T^+ = T - T_w/T_b - T_w$, eq. (18) becomes

$$\frac{d^2T^+}{dy^{+2}} = -u^+Nu_a \tag{22}$$

with boundary conditions

$$T^+(1) = 0, \quad T^{+'}(0) = 0 \tag{23}$$

Defining new dimensionless temperature, $T^{++} = T^+/Nu_a$, eq. (23) becomes

$$\frac{d^2T^{++}}{dy^{+2}} = -u^+ \tag{24}$$

with boundary conditions

$$T^{++}(1) = 0, \quad T^{++'}(0) = 0 \tag{25}$$

From the definition of bulk temperature,

$$T_B = \frac{\int_{A_c} uTdA_c}{A_c\bar{u}} = \frac{1}{\bar{u}a}\int_0^a uTdy \tag{26}$$

$$\int_0^1 u^+T^{++}dy^+ = \frac{1}{Nu_a} \tag{27}$$

Equations (24) and (27) can be solved numerically over the appropriate range of shear rate parameter ($\beta$) to determine the asymptotic Nusselt numbers for constant heat flux.

Next consider the case where the surface temperature is constant (CWT). From $\partial T/\partial x(T - T_w/T_b - T_w) = 0$ and $T_w$= constant,

$$\frac{\partial T}{\partial x} = \frac{T - T_w}{T_b - T_w}\frac{\partial T_b}{\partial x} \tag{28}$$

The dimensionless form of eq. (18) becomes

$$\frac{d^2T^+}{dy^{+2}} = -u^+T^+Nu_a \tag{29}$$

with boundary conditions

$$T^+(1) = 0, \quad T^{+'}(0) = 0 \tag{30}$$

Integrating eq. (30), then,

$$\int_0^1 \frac{\partial^2T^+}{\partial y^{+2}}dy^+ = -\int_0^1 u^+T^+Nu_a dy^+ = -Nu_a \tag{31}$$

$$\left.\frac{\partial T^+}{\partial y^+}\right|_{y^+=1} - \left.\frac{\partial T^+}{\partial y^+}\right|_{y^+=0} = T^{+'}(1) = -Nu_a \tag{32}$$

Equations (29) and (32) can be solved numerically over the appropriate range of shear rate parameter ($\beta$) to determine the asymptotic Nusselt numbers for constant wall temperature.

## NUMERICAL ANALYSIS

The governing momentum equation and the global continuity equation were solved numerically. Solution of the resulting system of simultaneous equations was accomplished using an alternating direction implicit method with successive overrelaxation.

The following algorithm was used:

### Velocity field

Step 1: Specify a value of $n$ and $\beta$:

$$n = 1.2, 1.1, 1.0, 0.9, 0.8, 0.7, 0.6, 0.5, 0.4$$

$$10^{-4} \leq \beta \leq 10^4$$

Step 2: Assume a velocity profile starting with $u^{++}(y^+) = 1/2(1 - y^{+2})$ for a Newtonian fluid. The Newtonian velocity profile may then be used as the initial velocity profile for the non-Newtonian modified power law (MPL) calculation.

Step 3: Calculate $\eta_a^+ (y^+)$ field by using the assumed velocity field.

Step 4: Solve eq. (15) for $u^{++}$ ($y^{+}$) and by using ADI (alternating direction implicit) method and obtain $f\text{Re}_M$. TDMA (tri-diagonal matrix algorithm) may be used for obtaining the velocity profile.
Step 5: Calculate a new $\eta_a^{+}$ from a new value of velocity field.
Step 6: Calculate a new $u^{++}(y^{+})$ and $f\,\text{Re}_M$.
Step 7: Compare the $f_a$ $\text{Re}_M$value with a value calculated in step 4.
Step 8: Use the new $f_a\text{Re}_M$ to calculate a new $u^{++}(y^{+})$ and $f_a\text{Re}_M$until convergence.
Step 9: Obtain $u^{++}(y^{+})$ field and $f_a\text{Re}_M$.

**Temperature field (CHF)**

Step 1: The velocity profile was used to obtain the temperature profile.
Step 2: Similar to velocity profile, TDMA was used to obtain the temperature profile.
Step 3: Simpson's rule was used for calculation of Nusselt number.

**Temperature field (CWT)**

Either the eigenvalue method or the Runge–Kutta method may be used to find $T^{+\prime}$ by iteration of $T^{+\prime}(0) = 0$ and $T^{+\prime}(1)$. By using the definition of the bulk temperature, eq. (32) can be derived. The temperature distribution and Nusselt number can be obtained for various $n$ and $\beta$.

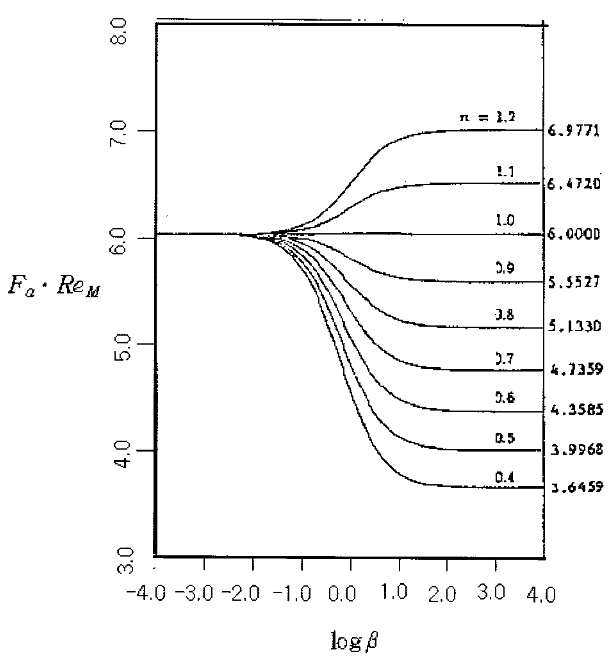


**Figure 3** Variation of the fully developed $f_a\text{Re}_M$ with $\beta$ and $n$

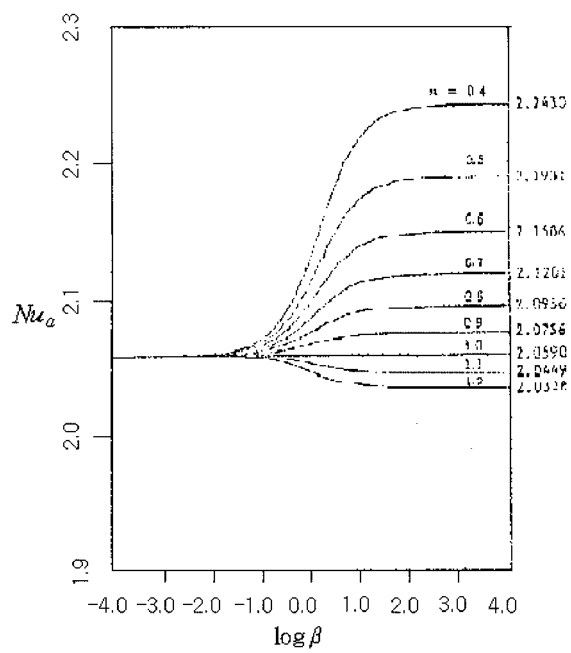


**Figure 4** Variation of the fully developed Nusselt numbers with $\beta$ and $n$ (CHF) .

## RESULTS AND DISCUSSION

Owing to the MPL constitutive equation, solutions were determined for flow between infinite parallel channels that are applicable over a wide shear rate range of pseudoplastic and dilatant fluids from Newtonian behavior at the lower shear rate range to power law behavior at the higher shear rate range. A shear rate parameter was identified that specifies whether a particular system for a typical pseudoplastic fluid is operating in the Newtonian, transition, or power law region. These results are shown in Figure 3 to Figure 5.

A numerical solution to eq. (7) for parallel channels is presented in Figure 3, which shows the quantitative relationship between pressure drop ($f\text{Re}_M$) and velocity gradient ($\beta$). As expected, at low $\beta$ values the solution approaches the Newtonian value and at large $\beta$ values the solution approaches that for a power law fluid. The shear rate parameter ($\beta$) reveals the three regions in the following manner: region I—Newtonian, $\beta \leq 10^{-2.5}$; region II —transition, $10^{-2.5}\ \beta \leq 10^{2.5}$; region III—power law, $\beta \geq 10^{-2.5}$. Thus the shear rate parameter $\beta$ can be used to determine in which of the three regions (Figure 1) a particular system is operating. As the shear rate parameter increases, the Reynolds number increases. As the power law flow index ($n$) approaches one, the tendency increases to retain Newtonian characteristics at low Reynolds numbers. As the flow index decreases, the tendency increases to retain the characteristics of power law fluid at high Reynolds numbers. Figure 3 illustrates the variation of the fully developed $f\mathbf{Re}_M$with $\beta$ and $n$ for the pseudoplastic and dilatant fluids through parallel channels.

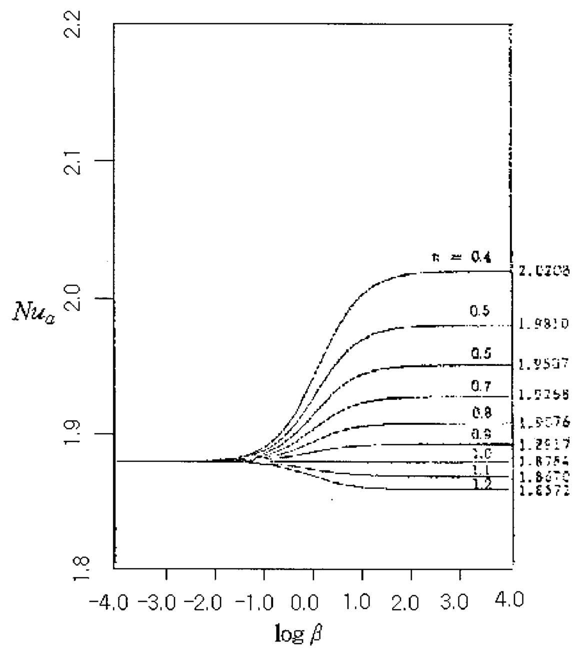


**Figure 5** Variation of the fully developed Nusselt numbers with $\beta$ and $n$ (CWT).

Figure 3 also expresses several important features of modified power law system. First, for complete similarity modeling, the modified Reynolds number $\mathrm{Re}_M$ and the parameter $\beta$ must both be considered. Also, a considerable difference exists if it is assumed that the system is operating in region III when it actually is operating in region I. Simple calculations show that errors in pressure drop predictions can occur as large as several hundred percent if such an uncertainty exists in the correct operating region. The fully developed flow with modified power law fluid for the thermal boundary conditions of CWT and CHF has been investigated for parallel channels. Thus the influence of the operating parameter $\beta$ on the fully developed Nusselt number is available. Figures 4 and 5 show the fully developed Nusselt numbers vs the shear rate parameter for parallel channels for the thermal boundary conditions of CWT and CHF. A comparison of Figures 3–5 reveals that influence of the shear rate parameter $\beta$ is much less for the heat transfer than for the pressure drop. In the latter case for the flow index ($n$) when $\beta$ changes from $10^{-4}$ to $10^{4}$ the product of the friction factor and the Reynolds number varies from 3.6459 to 6.9771, a difference of approximately 48%. On the other hand, for the same variation of $\beta$, the fully developed Nusselt number varies from 2.0328 to 2.2430 for the CHF condition and from 1.8572 to 2.0208 for the CWT condition with a difference of approximately 9 and 8%, respectively. Thus it would appear that the influence of $\beta$ on the hydrodynamic design is much more critical than for the thermal design. The optimum criteria for the best design of the heat transfer enhancement and the reduction in friction can be finally determined from present analysis.

For Newtonian fluid flow through parallel channels, the differences of $f_a\mathrm{Re}_M$and Nusselt number (CHF and CWT) between of the results of Shah and London[3] and the present results are less than 0.36%. These results are shown in Table I.

As expected, the numerical solutions at small values of $\beta$ approach the Newtonian analytical solutions.

$$f_a\mathrm{Re}_a = 6$$

$$Nu_{CHF} = 2.06$$

$$Nu_{CHT} = 1.88$$

The numerical solution at large values of $\beta$ in Figures 3–5 approach the following simple correlation of power law fluid analytical solutions (Irvine and Karni[1]):

$$f_a\mathrm{Re}_a = 2\left(\frac{1+2n}{n}\right)^n$$

$$Nu_{CHF} = 2.06\left(\frac{1+2n}{3n}\right)^{1/3}$$

$$Nu_{CWT} = 1.88\left(\frac{1+2n}{3n}\right)^{1/3}$$

## CONCLUSIONS

The purpose of these research was to present several fundamental considerations in the area of low Reynolds number non-Newtonian channel flows. A discussion was given of the classification of different types of non-Newtonian fluids such as pseudoplastic and dilatant fluids was examined in greater detail.

Generalized Reynolds numbers were discussed, and several that appear in the literature were defined and examined. Caution was again advised with regard to the several such dimensionless groups that appear in published rheological studies (Irvine and Karni,[1] R. A. Brewster and T. F. Irvine, Jr.,[20] M. Capobianchi and T. F. Irvine, Jr.[21]).

**TABLE I**
**Comparison of $f_a$ $\mathrm{Re}_M$, $Nu_{a,\mathrm{CHF}}$ and $Nu_{a,\mathrm{CWT}}$ of Newtonian Fluid in a Parallel Channel**

| | $f_a$ $\mathrm{Re}_M$ | $Nu_{\mathrm{a,CHF}}$ | $Nu_{\mathrm{a,CWT}}$ |
|---|---|---|---|
| (1)[a] | 24.000 | 8.235 | 7.541 |
| (2)[a] | 24.000 | 8.236 | 7.514 |

[a] (1) Shah and London.[3] (2) Present calculation.

The use of the generalized Reynolds number to predict the transition from Newtonian to non-Newtonian flow was considered and it was shown that such numbers are very sensitive to changes in the rheological property (flow index, $n$).

A method was outlined to predict fully developed laminar pressure drop and heat transfer in parallel channels for laminar non-Newtonian flow that utilize presently available solutions for laminar Newtonian flow.

By using a more general constitutive equation, the modified power law equation, solutions are possible that take this shear rate dependence into account and through a dimensionless shear rate parameter enable an appropriate choice of the pressure drop and heat transfer solutions.

Numerical solutions for laminar fully developed flow between infinite parallel channels are presented for the friction factor times Reynolds number and the Nusselt number (CHF and CWT) for a MPL fluid that has Newtonian behavior at low shear rates, power law behavior at high shear rates, and a transition region in between (Fig. 1). By using the MPL constitutive equation, solutions are obtained that are applicable over a wide shear rate range of pseudoplastic and dilatant fluids from the Newtonian behavior at a lower shear rate range to the power law behavior at a higher shear rate range.

A shear rate parameter can be used for the prediction of the shear rate range for a specified set of operating conditions. The shear rate parameter defines the transition region (approximately $10^{-2.5} \leq \beta \leq 10^{2.5}$) and is useful in determining whether the fluid is a fully developed Newtonian fluid ($\beta \leq 10^{-2.5}$) or a fully developed power law fluid ($\beta \geq 10^{2.5}$). A numerical solution reveals that serious errors can result if the pressure drop for a power law fluid applied when the fluid is actually operating in the transition region. Of particular interest are duct flows operating at low Reynolds numbers. As the shear rate increases, the tendency increases to retain power law fluid characteristics at low Reynolds numbers. As the shear rate decreases, the tendency increases to retain Newtonian fluid characteristics at high Reynolds numbers.

For pseudoplastic and dilatant non-Newtonian fluids, the MPL model is recommended for use because the fluid properties have big differences between the power law model and the actual values in low and medium range of shear rates.

The numerical solution enables the conservation of similitude when designing duct systems for such fluids as pseudoplastic and dilatant fluids since both the appropriate Reynolds and Nusselt number and the shear rate ranges are considered.

From a comparison of the numerical calculations between Newtonian and non-Newtonian fluid flow, it is evident that for the thermal boundary conditions (CHF and CWT) a non-Newtonian fluid with flow behavior index less than one gives a higher Nusselt number than a Newtonian fluid. Due to the reduction in frictional drag and the enhancement in heat transfer rates, MPL fluids seem to be more optimum fluids in heat exchanger and liquid cooling module for electronic packaging compared to Newtonian fluids. On the other hand, the use of appropriate MPL fluids may lead to heat transfer enhancement without the handling difficulties.

## NOMENCLATURE

| | |
|---|---|
| $a$ | one half of slot width (m) |
| $c_p$ | specific heat (J/kg K) |
| $f_a$ | Darcy friction factor $[2(dp/dx)(a/\rho\bar{u})^2]$ (—) |
| $h$ | heat transfer coefficient $[q_w/(T_w - T_b)]$ (W/m$^2$ K) |
| $k$ | thermal conductivity (W/m K) |
| $K$ | power law consistency (N s$^n$/m$^2$) |
| $n$ | power law flow index (—) |
| $Nu_a$ | Nusselt number ($ha/k$) (—) |
| $p$ | pressure (N/m$^2$) |
| $q_w$ | wall heat flux (W/m$^2$) |
| $Re_a$ | Newtonian Reynolds number $(\rho\,\bar{u}a/\eta_0)$(—) |
| $Re_g$ | power law Reynolds number $(\rho\bar{u}^{2-n}a^n/K)$(—) |
| $Re_M$ | modified Reynolds number $(\rho\bar{u}a/\eta^*)$(—) |
| $T$ | temperature (K) |
| $T^+$ | dimensionless temperature (—) |
| $T^{++}$ | dimensionless temperature (—) |
| $u$ | velocity in flow direction (m/s) |
| $\bar{u}$ | duct average velocity (m/s) |
| $u^+$ | dimensionless velocity in flow direction $(u/\bar{u})$ (—) |
| $u^{++}$ | dimensionless velocity in flow direction $[u^+/(f_a\mathrm{Re}_M)/2]$ (—) |
| $x$ | coordinate in flow direction (m) |
| $y$ | coordinate in flow transverse direction (m) |
| $y^+$ | dimensionless coordinate in flow transverse direction (—) |

### Greek symbols

| | |
|---|---|
| $\alpha$ | thermal diffusivity (m$^2$/s) |
| $\beta$ | shear rate parameter $[\beta = (\eta_0/K)(\bar{u}/a)^{1-n}]$ (—) |
| $\gamma$ | shear rate (1/s) |
| $\eta_a$ | viscosity $(\tau/\gamma)$ (N s/m$^2$) |
| $\eta_0$ | zero shear rate viscosity (N s/m$^2$) (—) |
| $\eta^*$ | reference viscosity $\beta$ $[\eta_0(1 + \beta)]$ (N s/m$^2$) |
| $\eta^+$ | dimensionless viscosity $(\eta_0/\eta^*)$ (—) |
| $\rho$ | fluid density (kg/m$^3$) |
| $\tau$ | shear stress (N/m$^2$) |

### Subscripts

| | |
|---|---|
| $a$ | slot width |
| $b$ | bulk temperature |

$g$ generalized Reynolds number
$M$ modified Reynolds number
$w$ wall condition

## Superscripts

$+$ dimensionless quantities
$++$ dimensionless quantities
$'$ derivative

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